EXERCISES [MAI 4.10]

BINOMIAL DISTRIBUTION

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a)

exactly 4 heads	0.273
exactly 3 heads	0.219
3, 4 or 5 heads	0.711
no heads	0.00391
always heads	0.00391
at most 2 heads	0.145
at least 3heads	0.855

(b)

E(X)	4	Var(X)	2

2. $np = 10$ and $np(1-p) = 6$ **.** Hence $10(1-p) = 6 \Leftrightarrow p = 0.4$ and $n = 25$

3. $B(n, p)$ with $n = 5$ and $p = \frac{1}{2}$

(a) P(X = 3) = 0.3125 … = 0.313

(b) P($X \geq 1$) = 0.969

4. $B(n, p)$ with $n = 7$ and $p = 0.18$

(a) P(X = 2) = 0.252

(b) P($X \geq 2$) = 0.368

5. $B(n, p)$ with $n = 100$ and $p = 0.04$

(a) mean = np = 100 × 0.04 = 4

(b) P(X = 6) = 0.105

(c) P($X \geq 1$) = 0.983

6. $X \sim$ B(100,0.02)

(a) $E(X)$ = 100 × 0.02 = 2

(b) (i) P(X = 3) = 0.182 (ii) P($X > 1$) = 0.597

7. $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $\qquad p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$

(a) $B(n, p)$ with $n = 8$, $p = \frac{1}{8}$

(i) p(one black) = P(X = 1) = 0.393 to 3 s.f. (ii) p(at least one black) = P($X \geq 1$) = 0.656

(b) 400 draws: expected number of blacks = $\frac{400}{8}$ = 50

8. $X \sim B(n, p)$ with $n = 5$ and $p = \frac{1}{3}$

Therefore P(X = 3) = 0.165

9. (a) Probability = 0.138

(b) Probability = $(0.6)^2 \times 0.4 = 0.144 \left(\text{or } \frac{18}{125}\right)$

10. (a) X is $B(10, 0.25)$
$E(X) = 10 \times 0.25 = 2.5$

(b) $P(X \le 2) = 0.526$

11. X is Binomial $n = 5$ $p = 0.4$
$P(X \le 3) = 0.913$ to 3 s.f.

12. (a) $B(n, p)$ with $n = 3,\ p = \frac{1}{3}$

(i) $P(X = 3) = 0.0370$ or $P(3H) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

(ii) $P(X = 3) = 0.222$ or $P(2H, 1T) = 3\left(\frac{1}{3}\right)^2 \frac{2}{3} = \frac{2}{9}$

(b) (i) expected number of heads = $np = \left(\frac{1}{3} \times 12\right) = 4$

(ii) 4 heads, so 8 tails
E(winnings) = $4 \times 10 - 8 \times 6$ (= 40 – 48) = –\$ 8

13. $B(n, p)$ with $n = 7,\ p = \frac{1}{5}$

$P(X = 4) = 0.0287$

$P(X \ge 4) = 0.0333$

14. $B(n, p)$ with $n = 20,\ p = \frac{1}{4}$

(a) $E(X) = 20 \times \frac{1}{4} = 5$

(b) $P(X = 5) = 0.202$ to 3 s.f.

(c) $P(X < 5) = 0.415$ to 3 s.f. [less than five means $P(X \le 4)$]

15. (a) P(all ten cells fail) = 0.107 (or 0.8^{10})

(b) (satellite is still operating at the end of one year if $X \ge 1$

$P(X \ge 1) = 0.893$ (or 1 – 0.107= 0.893)

16. (i) mean = $10 \times 0.4 = 4$

(ii) check $P(X = 3) = 0.214$, $P(X = 4) = 0.251$, $P(X = 5) = 0.201$ so mode = 4

(iii) variance = $10 \times 0.4 \times 0.6 = 2.4$

(iv) st. dev = $\sqrt{2.4} = 1.55$

17. (i) mean = $10 \times \frac{1}{4} = 2.5$

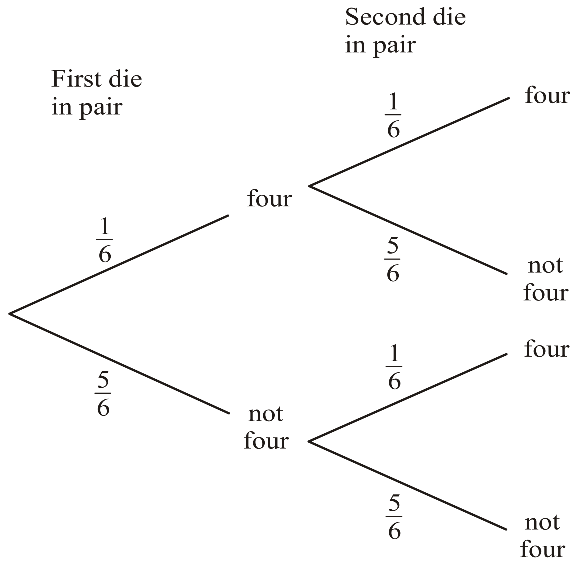
(ii) check $P(X = 2) = 0.281$, $P(X = 3) = 0.250$ so mode = 2

(iii) variance = $10 \times \frac{1}{4} \times \frac{3}{4} = \frac{15}{8} = 1.875$

(iv) st. dev = $\sqrt{1.875} = 1.37$

B. Paper 2 questions (LONG)

18. (a)



(b) $\mathrm{P}(E) = \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}\ \left(=\frac{5}{36}+\frac{5}{36}\right) = \frac{10}{36}\ \left(=\frac{5}{18} \text{ or } 0.278\right)$

(c) $X \sim \mathrm{B}\left(5, \frac{5}{18}\right)$

$\mathrm{P}(X=3) = 0.112$ [in fact $\binom{5}{3}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^2 = 0.112$]

(d) $\mathrm{P}(X \geq 3) = 0.135$

19. $B(n, p)$ with $n = 10$, $p = \frac{1}{4}$

(a) $\mathrm{E}(X) = 10 \times \frac{1}{4} = 2.5$

(b) $\mathrm{P}(X=6) = 0.0162$

(c) $\mathrm{P}(X \geq 2) = 0.756$

(d) Since $\mathrm{E}(X) = 2.5$ the mode is 2 or 3
Using GDC

x	$\mathrm{P}(X=x)$
1	0.188
2	0.282
3	0.250

From these values the most likely number of yellow ribbons is 2.

(e) The probability that a ribbon is yellow remains constant ($= \frac{1}{4}$)

20. $B(n, p)$ with $n = 20$, $p = 0.3$

(a) Mean $= 20\times0.3 = 6$ Variance $= 20\times0.3\times0.7 = 4.2$

(b) (i) $\mathrm{P}(X=5) = 0.179$ (ii) $\mathrm{P}(4 \leq X \leq 8) = 0.780$

(c) 0.3

(d) $0.7\times0.7\times0.3 = 0.147$